Coping with correlations in the analysis of the thermodynamic limit of neuronal networks

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Motivations:
The mathematical description and analysis of the thermodynamic limit of neuronal networks is important because it can shed light on the collective behaviors of large assemblies of neurons such as those which are found in the brains of mammals. There are several other reasons why it is worth investigating these questions. One is that the limit equations, albeit complicated, are much more concise than the collection of equations describing individual neurons in the network. This makes their analysis possible with the consequence that general laws can be formulated rigourously. Another reason is that the mathematical framework in which they are derived allows one to evaluate precisely under which hypotheses the results are valid and how changes in the hypotheses will impact them.

Previous Work:
This area of research has primarily been investigated using methods from theoretical physics as in the work of H. Sompolinsky et al. (field theory) (SCS88), N. Brunel (BH99) (Master equation, population based approaches), among many others. Only recently has this problem received attention from mathematicians. Building on previous work on the large deviations of weakly interacting diffusions, in particular that of A. Guionnet (Gui97), O. Moynot and M. Samuelides (MS02) have generalized the results of Sompolinsky et al. for rate neurons, but the fact is that the vast majority of these approaches assume that the synaptic weights are either known and constant or that, if they are random, they are statistically independent.

Our contribution:
We have developed a new method for establishing the thermodynamic limit of a network of fully connected rate neurons with correlated, Gaussian distributed, synaptic weights, and random inputs. The method is based on the formulation of a large deviation principle (LDP) for the probability distribution of the neuronal activity of a sequence of networks of increasing sizes. The motivation for using random connections comes from the fact that connections in neural networks are highly heterogeneous. The motivation for introducing correlation is the emphasis in computational modelling of neuroscience that neural networks are modular (i.e. the 'small worlds' literature), and the correlations in the connection distribution reproduce this modularity, unlike in (MS02; BFT15). The way this LDP is obtained is simpler than in (Gui97; MS02). It is also more general, as it leads to an annealed LDP that holds for all times. One can then obtain the limiting law of the empirical measure that holds for almost every interaction, for all time, unlike for example in (Gui97). The LDP allows one to prove that the limiting probability law is unique and to describe it completely by a McKean-Vlasov equation where the role of the correlations between the synaptic weights can be clearly assessed. Numerical results comparing the activity of the finite size network with the thermodynamic limit indicate that it is a very good approximation even for networks of modest size. The limiting probability law is Gaussian, and we perform a bifurcation analysis on the delay differential equations describing the mean and variance. This facilitates an understanding of the rich array of dynamics that neural networks with correlated synaptic weights can exhibit. Our new method is likely to be generalizable to large classes of neuron models, synaptic weights distributions, and network organizations.

References

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